MHD is beautiful
OK, if Berlusconi looses tonight
I am willing to give him a 25 h class
in serious MHD

on the other hand...
I have been challenged to show you that MHD can be more fun than some other stuff.
MHD is beautiful

MHD for observers / observations for MHD...

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Abstract: forget hand-waving arguments, field-waving is more fun!

my goals:

(nearly) no equations
(mostly) ugly cartoons
understand what MHD does
understand what MHD can't do
get a little but broad taste of the physics
know about numerical simulations

but above all... try to avoid headaches!
Euler equation

- motion of a point mass:
- for a fluid:

\[ m \frac{d\vec{V}}{dt} = \vec{F} \]
\[ \rho \frac{d\vec{V}}{dt} = \vec{F} \]

\( \rho = \text{mass density, g cm}^{-3} \)
\( F = \text{volumic force} \)

\[ = -\nabla p + \rho \vec{g} + \frac{1}{c} \vec{j} \times \vec{B} \]

pressure gradient

gravity

Lorentz force

Magneto-Hydrodynamics: ions + electrons treated as a single fluid submitted to usual forces + Lorentz

what happens in reality: 2 fluids, ions and electrons

- electrons are much lighter -> carry the current
- feel Lorentz force
- transfer it to the ions (which have most of the inertia) by collisions
Closing the system

\[ \rho \text{ is given from } \vec{V} \text{ by the continuity equation} \]
(conservation of mass):
\[ \frac{d\rho}{dt} = -\rho \vec{\nabla}.\vec{V} \]

then \( p \) by the equation of state, e.g. polytropic:
\[ p\rho^{-\gamma} = \text{Const.} \]

and \( \vec{B} \) is given by Ohm's law:
\[ \vec{E} + \frac{1}{c} \vec{V} \times \vec{B} = \eta \vec{j} \]

not over yet! Need to get \( \vec{E} \) and \( \vec{j} \) from \( \vec{B} \), by Maxwell's equations

\[ \text{electric field, transformed to the fluid frame} \]
\[ \text{resistivity} = \text{electron/ion friction} \]
Maxwell's equations

\[ \nabla \times \vec{B} = \frac{4\pi}{c} \vec{j} \quad (+ \text{displacement current if relativistic}) \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{induction equation} \]
Ideal MHD

just as collisions, viscosity, heat transport etc...

resistivity (= magnetic diffusivity) computed from first principles
(e-i collisions) is extremely weak -> can often be neglected

\[ \vec{E} + \frac{1}{c} \vec{V} \times \vec{B} = 0 \]

coupled with induction:

\[ \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \]

gives:

\[ \frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{V} \times \vec{B}) = 0 \]

no demonstration here but this implies:

frozen flux (gas and field lines move together)

conservation of magnetic topology
Already two examples of the role of "frozen flux"

1°) Parker instability
in a stratified, magnetized medium

if a flux tube starts to buckle

gas starts sliding down along B

the bottom becomes heavier
-> sinks further down

the top becomes lighter
-> buoyancy pushes it further up

-> exponential growth !

-> extraction of magnetic flux from the Sun, galaxies... accretion disks?
2°) Magneto-Rotational instability

in a differentially rotating disk

give fluid particles a radial kick

unmagnetized case: they retain their angular momentum

→ oscillations around their initial position at the epicyclic frequency \( \kappa \) (= \( \Omega \) in keplerian disk)

magnetized case: the fluid particles stay tied to their field line

→ retain their rotation frequency

(exchange angular momentum by magnetic stresses)

this particle rotates too slowly to fight gravity

→ tends to fall further in

→ the small initial motion is amplified

→ instability due ONLY to flux freezing
The Lorentz force

\[ \vec{F} = \vec{j} \times \vec{B} = \frac{1}{4\pi} (\vec{\nabla} \times \vec{B}) \times \vec{B} \]

a small miracle of vector algebra,

... or rather: the subtle working

of the inner consistency of the equations of physics,

gives:

\[ \vec{F} = -\vec{\nabla} \left( \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} (\vec{B} \cdot \vec{\nabla}) \vec{B} \]

magnetic pressure

magnetic tension (a tensor !)

a magnetic field line acts as an elastic rope
magnetic energy

2 forces -> 2 forms of energy:

- **magnetic pressure** \( \frac{B^2}{8\pi} \)

compared to thermal energy density by

\[ \beta = \frac{8\pi p}{B^2} \]

**NB:** in astrophysics, since it is often impossible to measure B, one commonly assumes **equipartition:** \( \beta \sim 1 \)

this is not stupid!

- e.g. interstellar medium: \( p_{\text{gas}} \sim p_{\text{mag}} \sim p_{\text{cosmic rays}} \)

  because turbulence involving the 3 works toward equipartition

  ...but not universal! e.g. solar corona

- **magnetic tension** (twisting of field lines)

  -> new types of waves to propagate these new forms of energy
MHD waves

In an ordinary fluid, only 2 types of perturbations:

- **sound waves** (propagate pressure perturbations) \( \rightarrow \)
  \[ \omega^2 = k^2 c_s^2 \]

- **vortices** (vorticity, entropy) \( \rightarrow \) no propagation

In MHD, 3 waves:

- **Alfvén wave**: propagates magnetic torsion
  \[ \omega^2 = k_{\parallel}^2 V_A^2 \]

  with
  \[ v_A^2 = \frac{B^2}{4\pi \rho} \]

  Note:
  \[ \beta = \frac{8\pi p}{B^2} = \frac{2c_s^2}{v_A^2} \]

  Equipartition \( \rightarrow \)
  \[ c_s \sim v_A \]

only torsion, no compression in the wave
... and 2 magnetosonic waves

easier to discuss in one limit, e.g. \( \beta \lesssim 1 \)

essentially:

- **fast magnetosonic wave**
  \[
  \omega^2 = k^2 (v_A^2 + c_S^2)
  \]
  propagates pressure (thermal + magnetic) in all directions

- **slow magnetosonic wave**
  \[
  \omega^2 \approx k_{||}^2 c_S^2
  \]
  equilibrates pressure (thermal + magnetic) along field lines \( \rightarrow \) maintains
  \[
  p + \frac{B^2}{8\pi} = Constant
  \]
  low B, high p
  high B, low p
magnetic energy

• magnetic energy is stored as pressure or torsion of field lines

• just as gradients of entropy, velocity, etc. this can cause instabilities = means to release the free energy

• thanks to the variety of waves, the magnetic field also provides new channels to release this free energy (remember the MRI, due to gradient of $\Omega$, i.e. gravitational energy but exists only thanks to flux freezing)

• $\rightarrow$ all the classical hydrodynamical instabilities (Rayleigh-Taylor, Kelvin-Helmholtz, etc.), modified by the magnetic field

• $\rightarrow$ new ones, due to the currents
MHD is all about where energy is,
where it can go,
how it can go there
and how it can be dissipated
to heat,
to radiation,
or to particle acceleration
e.g. kink instability

(applications to knots in jets)

a vertical field
+ a vertical current $\rightarrow$ a helicoidal field

if the current becomes too strong
(i.e. winds the field too much)

twisting of the whole configuration

as the elastic engine of a model airplane, when it can't take more torsion energy
Reconnection

MHD often presses together regions of opposite magnetic field

→ formation of current sheets

very strong $\mathbf{j}$

→ $\eta \mathbf{j}$ can act

→ reconnection:

→ change of topology:

Amari et al.: simulation of a CME
16 February 1980: White Light

Source: High Altitude Observatory Archives
magnetic energy and flux from inside the sun emerge in the corona (Parker). Energy stored in magnetic loops is suddenly released as kinetic energy (+ heat).
reconnection: a simple case
(tearing instability)

- ideal MHD often brings together regions of opposite B
- \( \rightarrow \) current sheets \( \rightarrow \) resistivity can act

\[
\frac{dB_y}{dx} = \frac{4\pi}{c} j_z
\]

(plus a constant Bz)

an ideal MHD instability compresses the field lines,
resistivity allows the formation of magnetic
islands \( \rightarrow \) destruction of magnetic flux
Reconnection (continued)

(need a few equations...)

- use a simple description:

\[ \vec{B} = \vec{e}_z \times \vec{\nabla} \psi(x, y, z) \]

\[ B_x = -\frac{\partial \psi}{\partial y} \]
\[ B_y = +\frac{\partial \psi}{\partial x} \]

- equilibrium:

\[ \psi(x, y, z) = B_0 \frac{x^2}{2} \rightarrow B_y = B_0 x \]

- note: \( \psi \) = constant along field lines

= magnetic flux (per unit length) below a field line

- from Ohm's law (induction equation)

\[ \vec{E} + \frac{1}{c} \vec{V} \times \vec{B} = \eta \vec{j} \]

one gets

\[ \frac{\partial \psi}{\partial t} + \vec{V} \cdot \vec{\nabla} \psi = \eta \Delta \psi \]

\[ \Rightarrow \frac{d\psi}{dt} = \eta \Delta \psi \]

local variation + variation along the gas trajectory = total variation with the gas flow
reconnection...

- ideal MHD: \[ \frac{d\psi}{dt} = 0 \] frozen magnetic flux

- resistive MHD: \[ \frac{d\psi}{dt} = \eta \Delta \psi \] diffusion of magnetic flux

\[ \eta = \text{resistivity} = \text{magnetic diffusivity}, \quad \text{extremely small in most plasmas} \]

\[ \Rightarrow \text{MHD has to produce extremely thin regions with very strong gradients} \]

\[ \Rightarrow \left( \frac{\partial}{\partial x} \sim \eta^{1/2} \right) \text{for resistivity to act} \Rightarrow \text{often very slow} \]

and extremely localized

One often invokes “turbulent resistivity”

just as turbulent viscosity

this is just as convenient

BUT EVEN MORE MISLEADING

(NEVER true in laboratory plasmas)
non-ideal MHD

• ideal MHD: \[ \vec{E} + \frac{1}{c} \vec{V} \times \vec{B} = 0 \Rightarrow E_\parallel = 0 \]

• resistive: \[ \vec{E} + \frac{1}{c} \vec{V} \times \vec{B} = \eta \vec{j} \Rightarrow E_\parallel \neq 0 \]

⇒ possibility to accelerate particles along field lines (if not too collisional)

⇒ e.g. solar flares

⇒ already non-MHD effects! (some) energy can go to particle acceleration (ordered motion) rather than heat

⇒ kinetic effects may affect the dynamics in the reconnection layer in a manner that differs strongly from resistivity

⇒ believing in turbulent viscosity is already bad manners

if you believe too much in turbulent resistivity,

don't pretend you haven't been warned!

lesson: if you want to be able to assess critically and constructively MHD results, the SECOND BEST WAY is to count a plasma theorist among your very best friends (the very best is to hire one)
auroras are the result of non-ideal MHD processes in the tail of the magnetosphere, accelerating particles along magnetic field lines. The theory (still incomplete) is probably even more beautiful than the real thing.
bad news...

"he" called and said he wasn't interested in the 25h MHD class

so I will have to dump that course on you before friday
unless we try with this guy?

new concept: the totally masked coded mask
turbulent dynamo

simple dynamo

self-excited dynamo generates its own B from any small ("seed") magnetic field

Turbulent dynamo (Earth, Sun, galaxies (?), accretion disks (?)):
If the turbulent velocity field has the right property ("helicity") it can generate a large-scale magnetic field

\[ \rightarrow \text{conversion of kinetic to magnetic energy} \]
Brun et al.: simulation of the solar dynamo
a word about MHD simulations

- problems due to the discretization:
  \[
  \frac{df}{dx} \to \frac{f(x + dx) - f(x)}{dx} = \frac{df}{dx} + (\ldots) f
  \]

- maintain \( \vec{\nabla} \cdot \vec{B} = 0 \) (some people don't! -> accretion on monopoles ?)
  -> constrained transport (ZEUS) still used in more modern codes
  but adds complexity; other methods...
- stability issues (since various waves, sometimes at very different speeds)
- resolving very small scales, boundary conditions, initial equilibrium...
- numerical "resistivity" and "viscosity" (more diffusive -> better movies!)
- more modern codes under development (VAC, Athena, Astro-Bear, Ramses-MHD...):
  Godounov algorithm (--> 2nd order precision, solves shocks and discontinuities...).
  BUT exact Godounov scheme for hydro, only approximate ones in MHD (NO universal one)
- Adaptive Mesh Refinement
dynamical disk instabilities

- need to explain / understand the turbulence causing accretion
- generally turbulence is due to external forced motion (e.g. an object moving in a fluid)
- or to instabilities: extract free energy from the fluid (due to gradients of density, entropy, to currents...) and transform it into kinetic energy (turbulent motion) -> thermalized as heat -> decreases the gradient
- e.g. convection transports heat better than diffusion
- example of a disk instability: galactic spirals, due to self-gravity
- but self-gravity is too weak in accretion disks
- => stuck for a long time
- 1991: Balbus and Hawley => Magneto-Rotational Instability
- (1990: Tagger et al.: magnetic spiral instability, but too weak)
- 1999: Tagger & Pellat, Accretion-Ejection Instability, (usefull) complement to MRI
- + much additional work...
Magneto-Rotational Instability

this perturbed motion releases

gravitational energy
\[ \propto \rho \frac{\partial}{\partial r} (\Omega^2) \]

but also costs magnetic energy to

bend the field lines
\[ \propto k_z^2 B^2 \propto \rho k_z^2 v_A^2 \]

\[ r \frac{\partial}{\partial r} (\Omega^2) + k_z^2 v_A^2 < 0 \]

\[ \beta = \frac{8\pi p}{B^2} > 1 \]

on the other hand \((k_z h)^2\) must be > 1 to fit in the disk, and (see King's lecture)

\( h \sim c_s / \Omega \). Using the expression for \( v_A^2 \) the criterion boils down to:

\[ \beta = \frac{8\pi p}{B^2} > 1 \]

\[ \text{-> any weakly magnetized disk!} \]
MRI (continued)

main successes:
• explains turbulence in any magnetic geometry, any magnetic field < equipartition
• causes accretion at reasonable rate (NOT $\alpha$ viscosity...)
• generates by itself strong magnetic field

• however:
  - jets ?
  - no QPO, even in GR – MHD
  - constraint on numerical simulations:
    no net vertical magnetic flux in the disk
numerical simulations of the MRI
(Hawley and co-workers)

local

Global 3D
Komissarov: axisymmetric simulation of the Blandford–Znajek and Penrose processes. B allows to extract energy from a spinning Black Hole ... but doesn't seem efficient enough for jets
If the disk is threaded by a vertical field

- because fluid particles have to move along field lines, and to rotate at their angular velocity
- project the forces along field lines =>
- centrifugal force wins if the field line is inclined more than 30° to the vertical (beads on a wire..., Henriksen)
- fluid particles are accelerated, while the whole field line still rotates at the same $\Omega$

-> allows to extract a lot of angular momentum with ejection of only a little gas, and self-collimation
self-collimation of MHD jets

- When the azimuthal velocity becomes larger than $\frac{r \Omega}{v_A}$
  - kinetic energy $>$ magnetic energy
  - the gas is able to bend the field lines
  - so that moving up limits its angular velocity (as on a corkscrew)

then $B_r \rightarrow B_\theta \rightarrow j_z \rightarrow \vec{J}_z \times \vec{B}_\theta$ (hoop stress)

$\Rightarrow$ SELF-COLLIMATION!

to explain the long, thin jets
magnetized accretion-ejection structures connecting disk to jet (Pelletier, Ferreira + coworkers)

(connection to the disk - => requires $\beta \sim 1$)

F. Casse

(axisymmetric simulation

-> no turbulence

-> using "anomalous resistivity" to allow continuous description from the disk to the jet, following analytical results
Quasi-Periodic Oscillations and normal modes

- some QPO models -> only predict frequencies
  -> have to assume orbiting "blobs"
- but blobs would be sheared away by differential rotation in ~ 1 rotation time
- whereas QPOs are (quasi-)coherent
- an alternative: normal modes = standing wave patterns (as in any cavity, i.e. waveguide, microwave oven, the Sun, a bell...) -> seismology
- but need for external excitation (hammer on the bell)
- whereas the LF-QPO can reach 40% RMS (BIG hammer needed!)
- best possibility: UNSTABLE NORMAL MODES as the (barred) spiral in Galaxies
- what is the best means to make them unstable?

the magnetic field!
Accretion–Ejection Instability

- A normal mode (as in seismology, waveguides, microwave ovens, the bridge on Saturday…) i.e. a single frequency standing wave pattern which is unstable i.e. self-excited, no need for external excitation

- in the conditions required by jet models (vertical field, $\beta \sim 1$), a spiral instability near the inner edge of the disk

- can redirect a significant fraction of the accretion energy upward as Alfvén wave (whence its name) though not a jet yet!

Tagger & Pellat, 1999
AEI (continued)

• with J. Rodriguez, P. Varnière: possible explanation for the low-frequency QPO

-> "Magnetic Floods" scenario for the cycles of GRS 1915+105:
-> controlled by the advection and cycling of vertical magnetic flux

Markwardt

Chaty (PhD thesis), Mirabel et al.
added bonus: the "forbidden" transition of Belloni et al.:

Magnetic floods scenario: cycles controlled by the cycling of vertical magnetic flux in the disk and the central hole.
2D numerical simulation of the AEI

(2D well adapted because thin disk, $n_z=0$ mode)

A spiral wave appears with initially 3 arms (depends on initial conditions...)
then 2 and finally 1 arm
As gas and magnetic flux accumulate in the inner region

Caunt & Tagger
The flares at the Galactic Center

A different but closely related instability (RWI: Lovelace et al.), in its MHD form:

If a gas blob joins the disk and circularizes at a few tens $r_{\text{Schwartzschild}}$

accretion rate at the inner edge, not unlike the IR and X-ray lightcurves (see Eckart's talk)

Tagger & Melia, 2006
next stage

- HF-QPO of BH binaries
- coming soon

I was supposed to write the paper here but the other talks are too interesting!